Network Coding and Decoding Technique for a Multiple Unicast Network with Linear Block Code Utilized as Network Codes

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Abstract – We propose a simple yet effective wireless network coding and decoding technique for a multiple unicast network. It utilizes spatial diversity through cooperation between nodes which carry out distributed encoding operations dictated by generator matrices of linear block codes. In order to exemplify the technique, we make use of greedy codes over the binary field and show that the arbitrary diversity orders can be flexibly assigned to nodes. Furthermore, we present the optimal detection rule for the given model that accounts for intermediate node errors and suggest a low-complexity network decoder using the sum-product (SP) algorithm. The proposed SP detector exhibits near optimal performance. We also show asymptotic superiority of network coding over a method that utilizes the wireless channeling a repetitive manner without network coding (NC) and give related rate-diversity trade-off curves. Finally, we extend the given encoding method through selective encoding in order to obtain extra coding gains.

Index Terms – Wireless network coding, cooperative communication, linear block code, sum-product decoding, unequal error protection.

1. INTRODUCTION

1.1. Network Coding

Network coding, as a field of study, is young. It was only in 2000 that the seminal paper by Ahlswede, Cai, Li, and Yeung [4], which is generally attributed with the "birth" of network coding, was published. As such, network coding, like many young fields, is characterized by some degree of confusion, of both excitement about its possibilities and skepticism about its potential. Clarifying this confusion is one of the principal aims of this book. Thus, we begin soberly, with a definition of network coding.

1.2. What is network coding?

Defining network coding is not straightforward. There are several definitions that can be and have been used. In their seminal paper [4], Ahlswede, Cai, Li, and Yeung say that they "refer to coding at a node in a network as network coding", where, by coding, they mean an arbitrary, causal mapping from inputs to outputs. This is the most general definition of network coding. But it does not distinguish the study of network coding from network, or multiterminal, information theory—a much older field with a wealth of difficult open problems. Since we do not wish to devote this book to network information theory (good coverage of network information theory already exists, for example.

1.3. Throughput

The most well-known utility of network coding and the easiest to illustrate is increase of throughput. This throughput benefit is achieved by using packet transmissions more efficiently, i.e., by communicating more information with fewer packet transmissions. The most famous example of this benefit was given by Ahlswede et al. [4], who considered the problem of multicast in a wire line network. Their example, which is commonly referred to as the butterfly network (see Figure 1.1). Features a multicast from a single source to two sinks, or destinations. Both sinks wish to know, in full, the message at the source node. In the capacitated network that they consider, the desired multicast connection can be established only if one of the intermediate nodes (i.e., a node that is neither source nor sink) breaks from the traditional routing paradigm of packet networks.

1.4. Multiple Unicast Transmissions

A recent approach, COPE, for improving the throughput of unicast traffic in wireless multi-hop networks exploits the broadcast nature of the wireless medium through opportunistic network coding. In this paper, we analyze throughput improvements obtained by COPE-type network coding in wireless networks from a theoretical perspective. We make two key contributions. First, we obtain a theoretical formulation for computing the throughput of network coding on any wireless network topology and any pattern of concurrent unicast traffic sessions. Second, we advocate that routing be made aware of network coding opportunities rather than, as in COPE, being oblivious to it. More importantly, our work studies the tradeoff between routing flows "close to each other" for utilizing coding opportunities and "away from each other" for avoiding wireless interference. Our theoretical formulation provides a method for computing source-destination routes and utilizing the best coding opportunities from available ones so as to maximize the throughput. We handle scheduling of broadcast transmissions subject to wireless transmit/receive diversity and link interference in our optimization framework. Using our formulations, we compare the performance of traditional unicast routing and network coding with coding-oblivious and coding-aware routing on a variety of mesh network topologies, including some derived from contemporary mesh network test beds. Our evaluations show that a route selection strategy that is aware of network coding opportunities leads to higher end-to-end throughput when compared to coding-oblivious routing strategies.

1.5. Unicast Transmissions

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2. LINEAR BLOCK CODES UTILIZED AS NETWORKCODES

2.1. Generator matrix

For generator matrices in probability theory, see matrix. In coding theory, a generator matrix is a matrix whose rows form a basis for a linear code. The code words are all of the linear combinations of the rows of this matrix, that is, the linear code is the row space of its generator matrix.

2.2. Terminology

If *G* is a matrix, it generates the code words of a linear code *C* by,

w = **s** *G*,

Where w is a code word of the linear code C, and s is any vector. A generator matrix for a linear $[n, k, d]_q$-code has format $k \times n$, where n is the length of a code word, k is the number of information bits (the dimension of C as a vector subspace), d is the minimum distance of the code, and q is size of the finite field, that is, the number of symbols in the alphabet (thus, q = 2 indicates a binary code, etc.). The number of redundant bits is denoted by r = n - k.

The standard form for a generator matrix is,[1]

$$G = \begin{bmatrix} I_k | P \end{bmatrix},$$

Where I_k is the k×k identity matrix and P is a k×r matrix. When the generator matrix is in standard form, the code C is systematic in its first k coordinate positions.[2]A generator matrix can be used to construct the parity check matrix for a code (and vice versa). If the generator matrix G is in standard form, $G = \begin{bmatrix} I_k | P \end{bmatrix}$, then the parity check matrix for C is[3]

$$H = \begin{bmatrix} -P^{\top} | I_{n-k} \end{bmatrix},$$

2.3. Generator Matrices and Parity Check Matrices

Definition 1.2.1 let C be an $[n,k]_q$ code. A **generator** matrix for C is any $k \times n$ matrix G with entries in $\mathbb{F}_q$ such that the rows of G form a basis for C.

Remark: If G is a generator matrix for C, then

$$C = \{\mathbf{x}G | \mathbf{x} \in \mathbb{F}_q^k\}$$

Definition 1.2.2 Let C be an $[n,k]_q$ code. Any set of k linearly independent columns of C is called an **information set** for C. More rigorously, Let C be an $[n,k]_q$ code and G a generator matrix for C. An information set for C is a set of integers $\{i_1, \ldots, i_n\} \subset \{1, \ldots, n\}$ such that the corresponding columns of G are linearly independent vectors in $\mathbb{F}_q^k$. (This is independent of the generator matrix)

Example: If C is an n-fold repetition code over $\mathbb{F}_q$. Then C is an $[n,1]_q$ code. Generator matrix: $[1, \ldots, 1]$

Every column forms (by itself) is an information set for C. Hence, we have n information sets for C.

Example: Let C be the code with the generator matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Then, C is an $[7,4]_2$ code. Information sets:

Where $P^\top$ is the transpose of the matrix P. This is a consequence of the fact that a parity check matrix of C is a generator matrix of the dual code $C^\perp$

2.4. Separation Vector as a Performance Metric

Our goal is now to explore the error performance metrics for network coding/decoding described in Section II. Our basic figure of merit will be the diversity order corresponding to giving information on the slope of decrease in logarithm of BER for *ui*, i.e., *P^^ui_=ui*for high SNR values. For conventional block coding, the average error performance over all data symbols is of interest.

2.5. An Example of Close-to-Optimal Linear Block Codes: Greedy Codes

In this study, we make use of some well-known linear block codes while constructing network codes that are to be used for the analysis of data rate and diversity orders for distinct symbols in Section III-C and simulation of BER in Section. However, the cooperative network coding described in this work and the resulting performance figures for a unicast pair are more general and applicable to any linear block code.

2.6. Theoretical Gains in Rate and Diversity for NC

In this section, we investigate the rate and diversity (asymptotic) gains of NC through use of the family of greedy network codes detailed in Section III-B, although the results are still valid for any other family of optimal or close-to optimal codes.

2.7. Sum-Product Network Decoder

The complexity of the optimal rule for decoding of any unicast transmission symbol ui grows exponentially, since the number of additions and multiplications in (10) increase exponentially in the number of users and transmissions. Therefore, this rule becomes inapplicable even for moderate-size networks.

3. RESULTS AND DISCUSSIONS

3.1. Numerical Results

3.1.1. Sample Network-I: Simulation Results

The results in this subsection are based on Sample Network-I of (1), consisting of 4 nodes, to observe the fundamental issues. At least 100 bit errors for each data bit *u*1, *u*2, and*u*3 are collected through simulations for each SNR value. In each run, data bits, intermediate node errors and complex channel gains are randomly generated with their probability distributions.

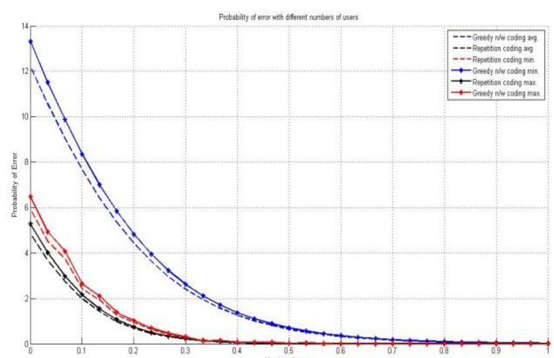

Figure 1 Probability density function

The performance determining parameter as the diversity order for individual source nodes is proposed for any given **G** over the corresponding separation vector. Through simulations we showed that our decoding rule, using reliability information for the network coded symbols, avoids the diversity order losses due to the error propagation effect. We presented design examples for network codes via greedy block codes, which may also provide unequal diversity orders to nodes with proper puncturing. Over given design examples, we obtained the rate-diversity trade-off curves and the rate advantage realized by using NC with respect to the no NC case. Moreover, the SP iterative network decoder with linear complexity order is proposed.

4. CONCLUSION

We formulated a NC problem for cooperative unicast transmissions. A generator matrix **G** and a scheduling vector **a**re used to represent the linear combinations performed at intermediate nodes. We presented a MAP-based decoding rule utilizing **G**, **v**, and the error probabilities at the intermediate nodes. A method for obtaining the performance determining parameter as the diversity order for individual source nodes is proposed for any given **G** over the corresponding separation vector. Through simulations we showed that our decoding rule, using reliability information for the network coded symbols, avoids the diversity order losses due to the error propagation effect. We presented design examples for network codes via greedy block codes, which may also provide unequal diversity orders to nodes with proper puncturing. Over given design examples, we obtained the rate-diversity trade-off curves and the rate advantage realized by using NC with respect to then NC case. Moreover, the SP iterative network decoder with linear complexity order is proposed.

REFERENCES

[1] Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity–part I and II: implementation aspects and performance analysis,” *IEEETrans. Commun.*, vol. 51, no. 11, pp. 1939–1948, Nov. 2003.
[2] J. Laneman, D. Tse, and G. Wornell, “Cooperative diversity in wireless networks: efficient protocols and outage behavior,” *IEEE Trans. Inf.Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
[3] D. Chen and J. Laneman, “Modulation and demodulation for cooperative diversity in wireless systems,” *IEEE Trans. Wireless Commun.*, vol. 5,no. 7, pp. 1785–1794, July 2006.
[4] F. Onat, A. Adinoyi, Y. Fan, H. Y. k¨omero˘glu, J. Thompson, and I. Marsland, “Threshold selection for SNR-based selective digital relaying incooperative wireless networks,” *IEEE Trans. Wireless Commun.*, vol. 57,pp. 4226–4237, Dec. 2008.
[5] T. Wang, A. Cano, G. Giannakis, and J. Laneman, “High-performance cooperative demodulation with decode-and-forward relays,” *IEEE Trans. Commun.*, vol. 55, pp. 1427–1438, July 2007.
[6] T. Wang, G. Giannakis, and R. Wang, “Smart regenerative relays for link-adaptive cooperative communications,” *IEEE Trans. Commun.*,vol. 56, pp. 1950–1960, Mar. 2008.
[7] S. Yang and R. Koetter, “Network coding over a noisy relay: a belief propagation approach,” in *Proc. 2007 IEEE International Symp. Inf.Theory*.
[8] R. Ahlswede, N. Cain, S. Li, and R. Yeung, “Network information flow,” *IEEE Trans. Inf. Theory*, vol. 46, pp. 1204–1216, July 2000.
[9] T. Ho, M. Medard, R. Koetter, D. Karger, M. Effros, J. Shi, and B. Leong, “A random linear network coding approach to multicast,”*IEEE Trans. Inf. Theory*, vol. 52, pp. 4413–4430, Oct. 2006.
[10] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft,“XORs in the air: practical wireless network coding,” *IEEE/ACM Trans.Netw.*, vol. 16, no. 3, pp. 497–510, June 2008.
[11] M. Di Renzo, M. Iezzi, and F. Graziosi, “Beyond routing via networkcoding: an overview of fundamental information-theoretic results,” in*Proc. IEEE Int. Symp. PIMRC*, pp. 2745–2750.
[12] M. Iezzi, M. Di Renzo, and F. Graziosi, “Network code design from unequal error protection coding: channel-aware receiver design and diversity analysis,” in *Proc. 2011 IEEE International Conf. Commun.*,pp. 1–6.
[13] M. Iezzi, M. D. Renzo, and F. Graziosi, *Flexible Network Codes Design for Cooperative Diversity, Adv. Trends in Wireless Commun.* InTech,2011.
[14] M. Xiao and T. Aulin, “Energy-efficient network coding for the noisy channel network,” in *Proc. 2006 IEEE International Symp. Inf. Theory*, pp. 778–782.
[15] G. O. Young, “Synthetic structure of industrial plastics (Book style with paper title and editor),” in *Plastics*, 2nd ed. vol. 3, J. Peters, Ed. New York: McGraw-Hill, 1964, pp. 15–64.

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